Before applying this file, please initialize Mathematica program!

Proposition 5.1.

In[1]:=
```
F1 := c0 x^2 + c2 y^2 + d0 x^3 + d1 x^2 y + d2 x y^2 +
  d3 y^3 + e0 x^4 + e1 x^3 y + e2 x^2 y^2 + e3 x y^3 + e4 y^4
```

In[2]:=
```
F2 := g11 x^2 + g22 y^2 + g33 z^2 + 2 g31 x z + 2 g32 y z +
  2 (x^2 + y^2 + z^2) (b1 x + b2 y + b3 z) + a (x^2 + y^2 + z^2)^2
```

In[3]:=
```
Coefficient[F2 /. {z → F1}, x^2] /. {x → 0, y → 0}
```

Out[3]= g11

In[4]:=
```
Coefficient[F2 /. {z → F1}, y^2] /. {x → 0, y → 0}
```

Out[4]= g22

In[5]:=
```
Coefficient[F2 /. {z → F1}, x^3] /. {x → 0, y → 0}
```

Out[5]= 2 b1 + 2 c0 g31

In[6]:=
```
Coefficient[F2 /. {z → F1}, x^2 y] /. {x → 0, y → 0}
```

Out[6]= $\frac{1}{2}$ (4 b2 + 4 c0 g32)

In[7]:=
```
Coefficient[F2 /. {z → F1}, x y^2] /. {x → 0, y → 0}
```

Out[7]= $\frac{1}{2}$ (4 b1 + 4 c2 g31)

In[8]:=
```
Coefficient[F2 /. {z → F1}, y^3] /. {x → 0, y → 0}
```

Out[8]= 2 b2 + 2 c2 g32

In[9]:=
```
Coefficient[F2 /. {z → F1}, x^4] /. {x → 0, y → 0}
```

Out[9]= a + 2 b3 c0 + 2 d0 g31 + $c0^2$ g33

In[10]:=
```
Coefficient[F2 /. {z → F1}, x^3 y] /. {x → 0, y → 0}
```

Out[10]= $\frac{1}{6}$ (12 d1 g31 + 12 d0 g32)

In[11]:=
```
Coefficient[F2 /. {z → F1}, x^2 y^2] /. {x → 0, y → 0}
```

Out[11]= $\frac{1}{4}$ (8 a + 8 b3 c0 + 8 b3 c2 + 8 d2 g31 + 8 d1 g32 + 8 c0 c2 g33)

In[12]:=
```
Coefficient[F2 /. {z → F1}, x y^3] /. {x → 0, y → 0}
```

Out[12]= $\frac{1}{6}$ (12 d3 g31 + 12 d2 g32)

In[13]:=
```
Coefficient[F2 /. {z → F1}, y^4] /. {x → 0, y → 0}
```

Out[13]= a + 2 b3 c2 + 2 d3 g32 + $c2^2$ g33

In[14]:=
```
Solve[{d0 == 2 b1 + 2 c0 g31, d2 == 1/2 (4 b1 + 4 c2 g31)}, {g31, b1}]
```

Out[14]= $\left\{\left\{g31 \to -\frac{-d0 + d2}{2\,(c0 - c2)},\ b1 \to -\frac{c2\,d0 - c0\,d2}{2\,(c0 - c2)}\right\}\right\}$

In[15]:=
$$\text{Solve}\left[\left\{d1 == \frac{1}{2}\,(4\,b2 + 4\,c0\,g32),\ d3 == 2\,b2 + 2\,c2\,g32\right\},\ \{g32,\ b2\}\right]$$

Out[15]=
$$\left\{\left\{g32 \to -\frac{-d1 + d3}{2\,(c0 - c2)},\ b2 \to -\frac{c2\,d1 - c0\,d3}{2\,(c0 - c2)}\right\}\right\}$$

e1 is

In[16]:=
$$\text{Simplify}\left[\frac{1}{6}\,(12\,d1\,g31 + 12\,d0\,g32)\ /.\ \left\{g31 \to -\frac{-d0 + d2}{2\,(c0 - c2)},\ g32 \to -\frac{-d1 + d3}{2\,(c0 - c2)}\right\}\right]$$

Out[16]=
$$-\frac{d1\,d2 + d0\,(-2\,d1 + d3)}{c0 - c2}$$

e3 is

In[17]:=
$$\text{Simplify}\left[\frac{1}{6}\,(12\,d3\,g31 + 12\,d2\,g32)\ /.\ \left\{g31 \to -\frac{-d0 + d2}{2\,(c0 - c2)},\ g32 \to -\frac{-d1 + d3}{2\,(c0 - c2)}\right\}\right]$$

Out[17]=
$$\frac{d1\,d2 + d0\,d3 - 2\,d2\,d3}{c0 - c2}$$

In[18]:=
$$\text{Solve}\left[\left\{e0 == a + 2\,b3\,c0 + 2\,d0\,g31 + c0^2\,g33,\ e2 == \frac{1}{4}\,(8\,a + 8\,b3\,c0 + 8\,b3\,c2 + 8\,d2\,g31 + 8\,d1\,g32 + 8\,c0\,c2\,g33),\ e4 == a + 2\,b3\,c2 + 2\,d3\,g32 + c2^2\,g33\right\}\ /.\ \left\{g31 \to -\frac{-d0 + d2}{2\,(c0 - c2)},\ g32 \to -\frac{-d1 + d3}{2\,(c0 - c2)}\right\},\ \{g33,\ b3,\ a\}\right]$$

Out[18]=
$$\left\{\left\{g33 \to -\frac{1}{(c0 - c2)^3}\left(d0^2 - d1^2 - 2\,d0\,d2 + d2^2 + 2\,d1\,d3 - d3^2 - c0\,e0 + c2\,e0 + c0\,e2 - c2\,e2 - c0\,e4 + c2\,e4\right),\right.\right.$$
$$b3 \to -\frac{1}{2\,(c0 - c2)^3}\left(-2\,c2\,d0^2 + c0\,d1^2 + c2\,d1^2 + c0\,d0\,d2 + 3\,c2\,d0\,d2 - c0\,d2^2 - c2\,d2^2 - 3\,c0\,d1\,d3 - c2\,d1\,d3 + 2\,c0\,d3^2 + 2\,c0\,c2\,e0 - 2\,c2^2\,e0 - c0^2\,e2 + c2^2\,e2 + 2\,c0^2\,e4 - 2\,c0\,c2\,e4\right),$$
$$\left.\left.a \to -\frac{1}{(c0 - c2)^3}\left(c2^2\,d0^2 - c0\,c2\,d1^2 - c0\,c2\,d0\,d2 - c2^2\,d0\,d2 + c0\,c2\,d2^2 + c0^2\,d1\,d3 + c0\,c2\,d1\,d3 - c0^2\,d3^2 - c0\,c2^2\,e0 + c2^3\,e0 + c0^2\,c2\,e2 - c0\,c2^2\,e2 - c0^3\,e4 + c0^2\,c2\,e4\right)\right\}\right\}$$

Proposition 5.2.

In[19]:=
$$\begin{aligned}P[T\_] :=\ & -d0^2 + \left(-4\,c0^4 + 4\,c0^3\,c2 - 6\,d0^2 + d1^2 + 2\,d0\,d2 + 4\,c0\,e0 - 4\,c2\,e0\right)T^2 + \\ & (-8\,d0\,d1 + 4\,d1\,d2 + 4\,d0\,d3 + 4\,c0\,e1 - 4\,c2\,e1)\,T^3 + \\ & \left(-12\,c0^3\,c2 + 12\,c0^2\,c2^2 - 5\,d0^2 - 2\,d1^2 - 4\,d0\,d2 + 3\,d2^2 + 6\,d1\,d3 + 4\,c0\,e0 - 4\,c2\,e0 + 4\,c0\,e2 - 4\,c2\,e2\right)T^4 + (-8\,d0\,d1 + 8\,d2\,d3 + 4\,c0\,e1 - 4\,c2\,e1 + 4\,c0\,e3 - 4\,c2\,e3)\,T^5 + \\ & \left(-12\,c0^2\,c2^2 + 12\,c0\,c2^3 - 3\,d1^2 - 6\,d0\,d2 + 2\,d2^2 + 4\,d1\,d3 + 5\,d3^2 + 4\,c0\,e2 - 4\,c2\,e2 + 4\,c0\,e4 - 4\,c2\,e4\right)T^6 + (-4\,d1\,d2 - 4\,d0\,d3 + 8\,d2\,d3 + 4\,c0\,e3 - 4\,c2\,e3)\,T^7 + \\ & \left(-4\,c0\,c2^3 + 4\,c2^4 - d2^2 - 2\,d1\,d3 + 6\,d3^2 + 4\,c0\,e4 - 4\,c2\,e4\right)T^8 + d3^2\,T^{10}\end{aligned}$$

In[20]:=
```
Collect[P[t] /. {e1 → (2 d0 d1 - d1 d2 - d0 d3) / (c0 - c2),
    e3 -> (2 d2 d3 - d1 d2 - d0 d3) / (c2 - c0)}, t, Simplify]
```

Out[20]=
$$-d0^2+\left(-4\,c0^4+4\,c0^3\,c2-6\,d0^2+d1^2+2\,d0\,d2+4\,c0\,e0-4\,c2\,e0\right)t^2+\left(-12\,c0^3\,c2+12\,c0^2\,c2^2-5\,d0^2-2\,d1^2-4\,d0\,d2+3\,d2^2+6\,d1\,d3-4\,c2\,e0-4\,c2\,e2+4\,c0\,(e0+e2)\right)t^4+\left(-12\,c0^2\,c2^2-3\,d1^2-6\,d0\,d2+2\,d2^2+4\,d1\,d3+5\,d3^2-4\,c2\,e2-4\,c2\,e4+4\,c0\left(3\,c2^3+e2+e4\right)\right)t^6+\left(-4\,c0\,c2^3+4\,c2^4-d2^2-2\,d1\,d3+6\,d3^2+4\,c0\,e4-4\,c2\,e4\right)t^8+d3^2\,t^{10}$$

Example 5.3.

In[21]:=
```
Solve[(x^2 + y^2 + z^2)^2 - 2 a x^2 - 2 b y^2 - 2 c z^2 + d^2 == 0, z]
```

Out[21]=
$$\left\{\left\{z\to-\sqrt{\left(c-x^2-y^2-\sqrt{\left(c^2-d^2+2\,a\,x^2-2\,c\,x^2+2\,b\,y^2-2\,c\,y^2\right)}\right)}\right\},\left\{z\to\sqrt{\left(c-x^2-y^2-\sqrt{\left(c^2-d^2+2\,a\,x^2-2\,c\,x^2+2\,b\,y^2-2\,c\,y^2\right)}\right)}\right\},\left\{z\to-\sqrt{\left(c-x^2-y^2+\sqrt{\left(c^2-d^2+2\,a\,x^2-2\,c\,x^2+2\,b\,y^2-2\,c\,y^2\right)}\right)}\right\},\left\{z\to\sqrt{\left(c-x^2-y^2+\sqrt{\left(c^2-d^2+2\,a\,x^2-2\,c\,x^2+2\,b\,y^2-2\,c\,y^2\right)}\right)}\right\}\right\}$$

In[22]:=
```
Series[Sqrt[1 + 2 t], {t, 0, 2}]
```

Out[22]=
$$1+t-\frac{t^2}{2}+O[t]^3$$

In[23]:=
```
Series[Sqrt[1 + t], {t, 0, 2}]
```

Out[23]=
$$1+\frac{t}{2}-\frac{t^2}{8}+O[t]^3$$

In[24]:=
```
F3[x_, y_] := s2 Sqrt(c - x^2 - y^2 + s1 L Sqrt(1 + 1 / L^2 (2 (a - c) x^2 + 2 (b - c) y^2)))
```

c0 is

In[25]:=
```
Simplify[D[F3[x, y], {x, 2}] / 2 /. {x → 0, y → 0}]
```

Out[25]=
$$-\frac{(L+(-a+c)\,s1)\,s2}{2\,L\,\sqrt{c+L\,s1}}$$

c2 is

In[26]:=
```
Simplify[D[F3[x, y], {y, 2}] / 2 /. {x → 0, y → 0}]
```

Out[26]=
$$-\frac{(L+(-b+c)\,s1)\,s2}{2\,L\,\sqrt{c+L\,s1}}$$

c0 - c2 is

In[27]:=
```
Simplify[-((L + (-a + c) s1) s2) / (2 L Sqrt[c + L s1]) - (-((L + (-b + c) s1) s2) / (2 L Sqrt[c + L s1]))]
```

Out[27]=
$$\frac{(a-b)\,s1\,s2}{2\,L\,\sqrt{c+L\,s1}}$$

e0 is

In[28]:=
```
Simplify[D[F3[x, y], {x, 4}] / 24 /. {x → 0, y → 0, s1 → 1}]
```

Out[28]=
$$\left(\left(-2\,(a-c)^2\,(c+L)-L\,(-a+c+L)^2\right)\,s2\right)\Big/\left(8\,L^3\,(c+L)^{3/2}\right)$$

In[29]:=
```
Simplify[D[F3[x, y], {x, 4}] / 24 /. {x → 0, y → 0, s1 → -1}]
```

Out[29]=
$$\frac{\left(2\,(a-c)^2\,(c-L)-L\,(a-c+L)^2\right)\,s2}{8\,(c-L)^{3/2}\,L^3}$$

e2 is

In[30]:=
```
Simplify[D[D[F3[x, y], {x, 2}] / 4, {y, 2}] /. {x → 0, y → 0, s1 → 1}]
```

Out[30]=
$$-\frac{1}{4\,L^3\,(c+L)^{3/2}}\left(a\left(2\,b\,c-2\,c^2+3\,b\,L-3\,c\,L-L^2\right)+(c+L)\left(2\,c^2+c\,L+L^2-b\,(2\,c+L)\right)\right)\,s2$$

In[31]:=
```
Simplify[D[D[F3[x, y], {x, 2}] / 4, {y, 2}] /. {x → 0, y → 0, s1 → -1}]
```

Out[31]=
$$\frac{1}{4\,(c-L)^{3/2}\,L^3}\left(a\left(2\,b\,c-2\,c^2-3\,b\,L+3\,c\,L-L^2\right)+(c-L)\left(2\,c^2-c\,L+L^2+b\,(-2\,c+L)\right)\right)\,s2$$

In[32]:=
```
Expand[-2 (a - c) (b - c) (c + L) - L (a - c - L) (b - c - L) +
  (a (2 b c - 2 c^2 + 3 b L - 3 c L - L^2) + (c + L) (2 c^2 + c L + L^2 - b (2 c + L)))]
```

Out[32]=
$$0$$

e4 is

In[33]:=
```
Simplify[D[F3[x, y], {y, 4}] / 24 /. {x → 0, y → 0, s1 → 1}]
```

Out[33]=
$$\left(\left(-2\,(b-c)^2\,(c+L)-L\,(-b+c+L)^2\right)\,s2\right)\Big/\left(8\,L^3\,(c+L)^{3/2}\right)$$

In[34]:=
```
Simplify[D[F3[x, y], {y, 4}] / 24 /. {x → 0, y → 0, s1 → -1}]
```

Out[34]=
$$\frac{\left(2\,(b-c)^2\,(c-L)-L\,(b-c+L)^2\right)\,s2}{8\,(c-L)^{3/2}\,L^3}$$

In[35]:=
```
Q[t_] := (t^2 + 1) (e0 + e2 t^2 + e4 t^4) - (c0 + c2 t^2)^3 /.
  {c0 -> -((L + (-a + c) s1) s2) / (2 L Sqrt[c + L s1]), c2 -> -((L + (-b + c) s1) s2) / (2 L Sqrt[c + L s1]),
   e0 -> ((-2 s1 (a - c)^2 (c + L s1) - L (-a + c + L s1)^2) s2) / (8 L^3 (c + L s1)^(3/2)), e2 ->
    1/(4 L^3 (c + L s1)^(3/2)) (-2 s1 (a - c) (b - c) (c + s1 L) - L (a - c - L s1) (b - c - L s1)) s2,
   e4 -> ((-2 s1 (b - c)^2 (c + L s1) - L (-b + c + L s1)^2) s2) / (8 L^3 (c + L s1)^(3/2))}
```

In[36]:=
```
Factor[Q[t] /. {s1 → 1, s2 → 1, L → Sqrt[c^2 - d^2]}]
```

Out[36]=
$$\left(\left(-a+c-b\,t^2+c\,t^2\right)\left(a-d+b\,t^2-d\,t^2\right)\left(a+d+b\,t^2+d\,t^2\right)\right)\Big/\left(8\left(c^2-d^2\right)^{3/2}\left(c+\sqrt{c^2-d^2}\right)^{3/2}\right)$$

In[37]:=
```
Factor[Q[t] /. {s1 → -1, s2 → 1, L → Sqrt[c^2 - d^2]}]
```

Out[37]=
$$-\left(\left(-a+c-b\,t^2+c\,t^2\right)\left(a-d+b\,t^2-d\,t^2\right)\left(a+d+b\,t^2+d\,t^2\right)\right)\Big/\left(8\left(c^2-d^2\right)^{3/2}\left(c-\sqrt{c^2-d^2}\right)^{3/2}\right)$$

In[38]:=
```
Factor[Q[t] /. {s1 → 1, s2 → -1, L → Sqrt[c^2 - d^2]}]
```

Out[38]=
$$-\left(\left(-a+c-b\,t^2+c\,t^2\right)\left(a-d+b\,t^2-d\,t^2\right)\left(a+d+b\,t^2+d\,t^2\right)\right)\Big/\left(8\left(c^2-d^2\right)^{3/2}\left(c+\sqrt{c^2-d^2}\right)^{3/2}\right)$$

In[39]:=
```
Factor[Q[t] /. {s1 → -1, s2 → -1, L → Sqrt[c^2 - d^2]}]
```

Out[39]=
$$\left(\left(-a+c-b\,t^2+c\,t^2\right)\left(a-d+b\,t^2-d\,t^2\right)\left(a+d+b\,t^2+d\,t^2\right)\right)\Big/\left(8\left(c^2-d^2\right)^{3/2}\left(c-\sqrt{c^2-d^2}\right)^{3/2}\right)$$

Proof of Theorem 2.10.

In[40]:=
```
NT := -5 RT KT^2 ET1 (RT DT - 2 (b T + a) CT^2) + DT^3 RT B1 + 2 DT^2 DT1 RT^2 B2 -
  DT^2 RT^2 KT ((3 d3 T + d2) (5 RT - (b^2 + 1) T^2) + (d1 T + 3 d0) (b^2 + 1)) +
  DT^2 B3 + 2 (b T + a) DT DT1 RT CT B4 + 10 (b T + a) DT DT2 RT^2 KT CT^2 +
  DT B5 - 4 (b T + a) DT1 RT KT CT^3 (5 (b T + a) CT1 + 2 b CT) +
  4 (b T + a) CT^4 KT (3 d0 B6 + d1 B7 + d2 B8 - 3 d3 T B9) + 4 CT^4 B10
```

In[41]:=
```
B1 := KT (42 (b^2 + 1) RT - 20 (a^2 + b^2 + 1)) -
  4 c2 RT (3 a b RT - 2 (a^2 + b^2 + 1 - a^2 b^2) T + 2 a b (a^2 + 1)) +
  2 (b^2 + 1) c1 RT (3 RT + 4 a b T + 4 a^2 + 4) - 4 (b^2 + 1) RT RT1 c0
```

In[42]:=
```
B2 := -(4 (b^2 + 1) T + 5 a b) KT - ((b^2 + 1) T^2 + a^2 + 1) KT1 +
  4 (a^2 + 1) (b^2 + 1) CT - 2 ((a^2 + 1) c2 + (b^2 + 1) c0) RT
```

In[43]:=
```
B3 := KT^2 (-4 a RT CT1 + 12 (b^2 + 1) (b T + a) T CT +
      8 a b (b T + a) CT + 12 b RT CT + 2 b RT (2 c0 + c1 T)) +
  KT KT1 (4 a b T (b T + a) CT - 12 (b T + a) RT CT + b T RT (2 c0 + c1 T) + a T RT CT1) +
  KT (-4 a (b^2 + 6) RT CT^2 - (18 (b^2 + 1) T (b T + a) + 16 a b (b T + a) - 4 b (a^2 + 1))
       RT CT CT1 + 8 (a^2 + 1) (b^2 + 1) (b T + a) CT^2 -
      8 (b T + a) ((a^2 + 1) c2 + (b^2 + 1) c0) RT CT + 4 a b T (b c1 - 2 a c2) RT CT +
      4 a (b^2 + 1) (2 c0 + c1 T) RT CT - 2 a c2 (2 c0 + c1 T) RT^2 - 2 b c0 CT1 RT^2 -
      12 (b^2 + 1) (b T + a) T RT1 CT^2 - 8 a b (b T + a) RT1 CT^2 - 4 (b T + a) RT RT1 CT CT1 +
      4 b RT RT1 CT^2 - 4 (b T + a) RT^2 CT1^2 - 8 c2 (b T + a) RT^2 CT + 4 b RT^2 CT CT1) -
  2 (b T + a) KT1 CT (2 a b T RT1 CT + 5 RT1 RT CT + 6 RT^2 CT1) -
  4 (b T + a) KT2 RT CT^2 (RT + a b T) -
  8 (a^2 + 1) (b^2 + 1) (b T + a) RT1 CT^3 +
  8 (b T + a) ((a^2 + 1) c2 + (b^2 + 1) c0) RT RT1 CT^2 -
  8 (a^2 + 1) (b^2 + 1) (b T + a) RT CT^2 CT1
```

In[44]:=
```
B4 := 2 (3 (b^2 + 1) T + 5 a b) KT CT + 5 RT KT CT1 + 2 ((b^2 + 1) T^2 + a^2 + 1) KT1 CT -
  8 (a^2 + 1) (b^2 + 1) CT^2 + 4 RT CT ((a^2 + 1) c2 + (b^2 + 1) c0)
```

In[45]:=
```
B5 := 9 CT^2 KT^3 + KT^2 CT^2 (6 RT1 CT + 4 RT CT1 + 8 b (b T + a) CT - 8 (b T + a)^2 CT1) -
  8 (b T + a)^2 KT KT1 CT^3 +
  KT CT^2 (-8 (b T + a) (b c1 - 2 a c2) RT CT - 8 b (b T + a) RT CT1 CT + 8 b^2 RT CT^2 -
     4 b (b T + a) RT CT CT1 - 48 (b T + a)^2 CT ((a^2 - 1) c2 + (b^2 + 1) c0) +
     24 a c1 (b T + a) RT1 CT - 48 (b T + a) CT (2 a (c2 - c0) - b c1) +
     12 (b T + a)^2 RT CT1^2 - 8 b (b T + a) RT1 CT^2 + 8 (b T + a)^2 RT1 CT CT1 +
     16 c2 (b T + a)^2 RT CT - 8 b (b T + a) RT CT CT1) +
  8 (b T + a)^2 KT1 CT^3 (RT1 CT + 3 RT CT1) + 8 (b T + a)^2 RT CT^4 KT2 -
  16 b (b T + a) RT CT^4 ((a^2 - 1) c2 + (b^2 + 1) c0) -
  32 (b T + a)^2 RT CT^3 CT1 ((a^2 - 1) c2 + (b^2 + 1) c0) +
  16 a (b T + a) RT RT1 c1 CT^3 CT1 + 4 a b RT RT1 c1 CT^4 +
  8 a (b^2 + 1) (b T + a) RT c1 CT^4 - 8 b RT CT^4 (2 a (c2 - c0) - b c1) -
  32 (b T + a) RT CT^3 CT1 (2 a (c2 - c0) - b c1)
```

In[46]:=
```
B6 := -(b^2 + 1) (b T + a) + 2 a
```

In[47]:=
```
B7 := -(b^2 + 1) T (b T + a) + a RT1 + 2 (a T + b)
```

In[48]:=
```
B8 := -(a^2 - 3) (b T + a) + a T RT1 - 4 a - 4 (b T + a) RT
```

In[49]:=
```
B9 := (a^2 - 1) (b T + a) + 2 a + 4 (b T + a) RT
```

In[50]:=
```
B10 := KT^2 (2 b CT - 4 (b T + a) CT1) - 2 (b T + a) CT KT KT1 +
  KT (a RT1 c1 CT - 4 (b T + a) CT ((a^2 - 1) c2 + (b^2 + 1) c0) - 4 a (c2 - c0) CT +
     2 b c1 CT - 2 a (b T + a)^2 c2 (2 c0 + c1 T) - 2 b (b T + a)^2 c0 CT1 +
     2 a b (b T + a) c1 CT + a (b T + a)^2 c1 CT1 + (1 / 2) (b T + a) RT1 c1 (2 c0 + c1 T) -
     2 (b T + a) (c2 - c0) (2 c0 + c1 T) + (b T + a) c1 CT1) +
  8 b (b T + a)^2 ((a^2 - 1) c2 + (b^2 + 1) c0) CT^2 +
  16 (b T + a)^3 CT CT1 ((a^2 - 1) c2 + (b^2 + 1) c0) -
  8 a (b T + a)^2 RT1 c1 CT CT1 - 2 a b (b T + a) RT1 c1 CT^2 -
  4 a (b^2 + 1) (b T + a)^2 c1 CT^2 + 4 b (b T + a) CT^2 (2 a (c2 - c0) - b c1) +
  16 (b T + a)^2 CT CT1 (2 a (c2 - c0) - b c1)
```

In[51]:=
```
AT1 := {-(T^2 + 1)^2 ET1 + 2 T CT^2 (2 CT + c0 - c2)} (4 ET - T ET1) +
  4 c0 CT^2 {(T^2 + 1) ET1 - 4 c2 T CT^2}
```

In[52]:=
```
AT2 := ET1 {-(T^2 + 1)^2 ET1 + 2 T CT^2 (2 CT + c0 - c2)} +
  4 c2 T CT^2 {(T^2 + 1) ET1 - 4 c2 T CT^2}
```

In[53]:=
```
NTR := 20 (c0 - c2)^2 T^2 (AT1 x + AT2 y)
```

In[54]:=
```
M := {{NT - NTR /. {KT -> RT1 CT - RT CT1,
        KT1 -> 2 (b^2 + 1) CT - 2 c2 RT, KT2 -> 2 (b^2 + 1) CT1 - 2 c2 RT1}} /.
    {RT -> (b^2 + 1) T^2 + 2 a b T + a^2 + 1, RT1 -> 2 (b^2 + 1) T + 2 a b,
     RT2 -> 2 (b^2 + 1), CT -> c0 + c1 T + c2 T^2, CT1 -> c1 + 2 c2 T, CT2 -> 2 c2,
     DT -> d0 + d1 T + d2 T^2 + d3 T^3, DT1 -> d1 + 2 T d2 + 3 d3 T^2, DT2 -> 2 d2 + 6 d3 T,
     ET1 -> e1 + 2 e2 T + 3 e3 T^2 + 4 e4 T^3, ET -> e0 + e1 T + e2 T^2 + e3 T^3 + e4 T^4}} /.
  {a -> 2 c0 x, b -> 2 c2 y, c1 -> 0, d0 -> 4 e0 x + e1 y, d1 -> 3 e1 x + 2 e2 y,
   d2 -> 2 e2 x + 3 e3 y, d3 -> e3 x + 4 e4 y}
```

In[55]:=
```
Simplify[M /. {x -> 0, y -> 0}]
```

Out[55]=
```
{{{0}}}
```

In[56]:=
```
Simplify[Coefficient[M, x, 1] /. {y -> 0}]
```

Out[56]=
```
{{{0}}}
```

In[57]:=
```
Simplify[Coefficient[M, y, 1] /. {x -> 0}]
```

Out[57]=
```
{{{0}}}
```

For AT = AT1 / (2 (c0 - c2))

In[58]:= q0 := e0 - c0^3

In[59]:= q1 := e1

In[60]:= q2 := e2 + e0 - 3 c0^2 c2

In[61]:= q3 := e3 + e1

In[62]:= q4 := e4 + e2 - 3 c0 c2^2

In[63]:= q5 := e3

In[64]:= q6 := e4 - c2^3

In[65]:= ga01 := -q1 / q0

In[66]:= ga11 := -q2 / q0

In[67]:= ga21 := -q3 / q0

In[68]:= ga31 := -q4 / q0

In[69]:= ga41 := -q5 / q0

In[70]:= ga51 := -q6 / q0

In[71]:= ga02 :=
-(q1 + q2 i + q3 i^2 + q4 i^3 + q5 i^4 + q6 i^5) / (q0 - q2 + q4 - q6 + (q1 - q3 + q5) i)

In[72]:= $$ga12 := -\frac{q2 + q3\,i + q4\,i^2 + q5\,i^3 + q6\,i^4}{q0 - q2 + q4 - q6 + (q1 - q3 + q5)\,i}$$

In[73]:= $$ga22 := -\frac{q3 + q4\,i + q5\,i^2 + q6\,i^3}{q0 - q2 + q4 - q6 + (q1 - q3 + q5)\,i}$$

In[74]:= $$ga32 := -\frac{q4 + q5\,i + q6\,i^2}{q0 - q2 + q4 - q6 + (q1 - q3 + q5)\,i}$$

In[75]:= $$ga42 := -\frac{q5 + q6\,i}{q0 - q2 + q4 - q6 + (q1 - q3 + q5)\,i}$$

In[76]:= $$ga52 := -\frac{q6}{q0 - q2 + q4 - q6 + (q1 - q3 + q5)\,i}$$

In[77]:= ga03 :=
-(q1 - q2 i + q3 i^2 - q4 i^3 + q5 i^4 - q6 i^5) / (q0 - q2 + q4 - q6 - (q1 - q3 + q5) i)

In[78]:= $$ga13 := -\frac{q2 - q3\,i + q4\,i^2 - q5\,i^3 + q6\,i^4}{q0 - q2 + q4 - q6 - (q1 - q3 + q5)\,i}$$

In[79]:= $$ga23 := -\frac{q3 - q4\,i + q5\,i^2 - q6\,i^3}{q0 - q2 + q4 - q6 - (q1 - q3 + q5)\,i}$$

In[80]:= $$ga33 := -\frac{q4 - q5\,i + q6\,i^2}{q0 - q2 + q4 - q6 - (q1 - q3 + q5)\,i}$$

In[81]:= $$ga43 := -\frac{q5 - q6\,i}{q0 - q2 + q4 - q6 - (q1 - q3 + q5)\,i}$$

In[82]:=
$$ga53 := -\frac{q6}{q0 - q2 + q4 - q6 - (q1 - q3 + q5)\,i}$$

In[83]:=
```
A1 := (AT1 /. {CT -> c0 + c2 T^2,
     ET1 → e1 + 2 e2 T + 3 e3 T^2 + 4 e4 T^3, ET → e0 + e1 T + e2 T^2 + e3 T^3 + e4 T^4})
```

In[84]:=
```
A2 := (AT2 /. {CT -> c0 + c2 T^2,
     ET1 → e1 + 2 e2 T + 3 e3 T^2 + 4 e4 T^3, ET → e0 + e1 T + e2 T^2 + e3 T^3 + e4 T^4})
```

In[85]:=
```
b11 := (A1 /. {T → 0}) / (2 (c0 - c2))
```

In[86]:=
```
b12 := -(A1 /. {T → i}) / (4 (c0 - c2))
```

In[87]:=
```
b13 := -(A1 /. {T → -i}) / (4 (c0 - c2))
```

In[88]:=
```
b21 := (A2 /. {T → 0}) / (2 (c0 - c2))
```

In[89]:=
```
b22 := -(A2 /. {T → i}) / (4 (c0 - c2))
```

In[90]:=
```
b23 := -(A2 /. {T → -i}) / (4 (c0 - c2))
```

In[91]:=
```
BT1 := (A1 /. {T → t}) / (2 (c0 - c2) t (t^2 + 1)) - b11 / t - b12 / (t - i) - b13 / (t + i)
```

In[92]:=
```
BT2 := (A2 /. {T → t}) / (2 (c0 - c2) t (t^2 + 1)) - b21 / t - b22 / (t - i) - b23 / (t + i)
```

In[93]:=
```
Cancel[Simplify[BT1]]
```

Out[93]=
$$\left\{\frac{1}{2\,(c0 - c2)}\right.$$
$$\left(-64\,c0^4\,c2^2 + 96\,c0^3\,c2^3 - 64\,c0^2\,c2^4 + 16\,c0\,c2^5 + 64\,c0^2\,c2\,e0 - 72\,c0\,c2^2\,e0 + 24\,c2^3\,e0 - 3\,e1^2 + 20\,c0^3\,e2 - 36\,c0^2\,c2\,e2 + 36\,c0\,c2^2\,e2 - 12\,c2^3\,e2 - 8\,e0\,e2 + 56\,c0^2\,c2\,e1\,t - 54\,c0\,c2^2\,e1\,t + 18\,c2^3\,e1\,t - 4\,e0\,e1\,t - 8\,e1\,e2\,t + 18\,c0^3\,e3\,t - 18\,c0^2\,c2\,e3\,t + 18\,c0\,c2^2\,e3\,t - 6\,c2^3\,e3\,t - 12\,e0\,e3\,t - 96\,c0^3\,c2^3\,t^2 + 64\,c0^2\,c2^4\,t^2 - 16\,c0\,c2^5\,t^2 + 56\,c0\,c2^2\,e0\,t^2 - 24\,c2^3\,e0\,t^2 - 3\,e1^2\,t^2 + 48\,c0^2\,c2\,e2\,t^2 - 36\,c0\,c2^2\,e2\,t^2 + 12\,c2^3\,e2\,t^2 - 8\,e0\,e2\,t^2 - 4\,e2^2\,t^2 - 10\,e1\,e3\,t^2 + 16\,c0^3\,e4\,t^2 - 16\,e0\,e4\,t^2 + 46\,c0\,c2^2\,e1\,t^3 - 18\,c2^3\,e1\,t^3 - 8\,e1\,e2\,t^3 + 40\,c0^2\,c2\,e3\,t^3 - 18\,c0\,c2^2\,e3\,t^3 + 6\,c2^3\,e3\,t^3 - 12\,e0\,e3\,t^3 - 8\,e2\,e3\,t^3 - 12\,e1\,e4\,t^3 - 64\,c0^2\,c2^4\,t^4 + 16\,c0\,c2^5\,t^4 + 16\,c2^3\,e0\,t^4 + 36\,c0\,c2^2\,e2\,t^4 - 12\,c2^3\,e2\,t^4 - 4\,e2^2\,t^4 - 10\,e1\,e3\,t^4 - 3\,e3^2\,t^4 + 32\,c0^2\,c2\,e4\,t^4 - 16\,e0\,e4\,t^4 - 8\,e2\,e4\,t^4 + 12\,c2^3\,e1\,t^5 + 26\,c0\,c2^2\,e3\,t^5 - 6\,c2^3\,e3\,t^5 - 8\,e2\,e3\,t^5 - 12\,e1\,e4\,t^5 - 4\,e3\,e4\,t^5 - 16\,c0\,c2^5\,t^6 + 8\,c2^3\,e2\,t^6 - 3\,e3^2\,t^6 + 16\,c0\,c2^2\,e4\,t^6 - 8\,e2\,e4\,t^6 + 4\,c2^3\,e3\,t^7 - 4\,e3\,e4\,t^7\right)\Bigg\}$$

In[94]:=
```
Rem1 := PolynomialRemainder[
  1/(2 (c0 - c2)) (-64 c0^4 c2^2 + 96 c0^3 c2^3 - 64 c0^2 c2^4 + 16 c0 c2^5 + 64 c0^2 c2 e0 -
     72 c0 c2^2 e0 + 24 c2^3 e0 - 3 e1^2 + 20 c0^3 e2 - 36 c0^2 c2 e2 + 36 c0 c2^2 e2 -
     12 c2^3 e2 - 8 e0 e2 + 56 c0^2 c2 e1 t - 54 c0 c2^2 e1 t + 18 c2^3 e1 t - 4 e0 e1 t -
     8 e1 e2 t + 18 c0^3 e3 t - 18 c0^2 c2 e3 t + 18 c0 c2^2 e3 t - 6 c2^3 e3 t - 12 e0 e3 t -
     96 c0^3 c2^3 t^2 + 64 c0^2 c2^4 t^2 - 16 c0 c2^5 t^2 + 56 c0 c2^2 e0 t^2 - 24 c2^3 e0 t^2 -
     3 e1^2 t^2 + 48 c0^2 c2 e2 t^2 - 36 c0 c2^2 e2 t^2 + 12 c2^3 e2 t^2 - 8 e0 e2 t^2 - 4 e2^2 t^2 -
     10 e1 e3 t^2 + 16 c0^3 e4 t^2 - 16 e0 e4 t^2 + 46 c0 c2^2 e1 t^3 - 18 c2^3 e1 t^3 - 8 e1 e2 t^3 +
     40 c0^2 c2 e3 t^3 - 18 c0 c2^2 e3 t^3 + 6 c2^3 e3 t^3 - 12 e0 e3 t^3 - 8 e2 e3 t^3 - 12 e1 e4 t^3 -
     64 c0^2 c2^4 t^4 + 16 c0 c2^5 t^4 + 16 c2^3 e0 t^4 + 36 c0 c2^2 e2 t^4 - 12 c2^3 e2 t^4 - 4 e2^2 t^4 -
     10 e1 e3 t^4 - 3 e3^2 t^4 + 32 c0^2 c2 e4 t^4 - 16 e0 e4 t^4 - 8 e2 e4 t^4 + 12 c2^3 e1 t^5 +
     26 c0 c2^2 e3 t^5 - 6 c2^3 e3 t^5 - 8 e2 e3 t^5 - 12 e1 e4 t^5 - 4 e3 e4 t^5 - 16 c0 c2^5 t^6 +
     8 c2^3 e2 t^6 - 3 e3^2 t^6 + 16 c0 c2^2 e4 t^6 - 8 e2 e4 t^6 + 4 c2^3 e3 t^7 - 4 e3 e4 t^7),
  q0 + q1 t + q2 t^2 + q3 t^3 + q4 t^4 + q5 t^5 + q6 t^6, t]
```

Therefore g0 is

In[95]:=
```
Simplify[Coefficient[Rem1, t, 0] + b11 ga01 + b12 ga02 + b13 ga03]
```

Out[95]=
```
{(c2^3 (24 e0^2 + e1^2 - 12 e0 e2) + e0 e3^2 - 24 c0^2 c2 e0 (c2^3 - e4) - 24 e0^2 e4 - e1^2 e4 +
    12 e0 e2 e4 + c0^3 (12 c2^3 e2 - e3^2 - 12 e2 e4)) / (2 (c0 - c2) (c2^3 - e4))}
```

In[96]:=
```
g0 := {(c2^3 (24 e0^2 + e1^2 - 12 e0 e2) + e0 e3^2 - 24 c0^2 c2 e0 (c2^3 - e4) - 24 e0^2 e4 - e1^2 e4 +
    12 e0 e2 e4 + c0^3 (12 c2^3 e2 - e3^2 - 12 e2 e4)) / (2 (c0 - c2) (c2^3 - e4))}
```

g1 is

In[97]:=
```
Simplify[Coefficient[Rem1, t, 1] + b11 ga11 + b12 ga12 + b13 ga13]
```

Out[97]=
```
{(e1 e3^2 + 2 c2^3 (21 e0 e1 - 4 e1 e2 - 7 e0 e3) - 26 c0^2 c2 e1 (c2^3 - e4) + 14 c0^3 e3 (c2^3 - e4) -
    42 e0 e1 e4 + 8 e1 e2 e4 + 14 e0 e3 e4) / (2 (c0 - c2) (c2^3 - e4))}
```

In[98]:=
```
g1 := {(e1 e3^2 + 2 c2^3 (21 e0 e1 - 4 e1 e2 - 7 e0 e3) - 26 c0^2 c2 e1 (c2^3 - e4) +
    14 c0^3 e3 (c2^3 - e4) - 42 e0 e1 e4 + 8 e1 e2 e4 + 14 e0 e3 e4) / (2 (c0 - c2) (c2^3 - e4))}
```

g2 is

In[99]:=
```
Simplify[Coefficient[Rem1, t, 2] + b11 ga21 + b12 ga22 + b13 ga23]
```

Out[99]=
```
{(e0 e3^2 + e2 e3^2 - 32 c0 c2^2 e0 (c2^3 - e4) - 19 e1^2 e4 - 24 e0 e2 e4 + 8 e2^2 e4 + 8 e1 e3 e4 +
    16 c0^3 (c2^3 - e4) e4 + 16 e0 e4^2 + c2^3 (19 e1^2 + 24 e0 e2 - 8 e2^2 - 8 e1 e3 - 16 e0 e4) +
    c0^2 c2 (8 c2^3 e2 - 3 e3^2 - 8 e2 e4)) / (2 (c0 - c2) (c2^3 - e4))}
```

In[100]:=
```
g2 := {(e0 e3^2 + e2 e3^2 - 32 c0 c2^2 e0 (c2^3 - e4) - 19 e1^2 e4 - 24 e0 e2 e4 + 8 e2^2 e4 + 8 e1 e3 e4 +
    16 c0^3 (c2^3 - e4) e4 + 16 e0 e4^2 + c2^3 (19 e1^2 + 24 e0 e2 - 8 e2^2 - 8 e1 e3 - 16 e0 e4) +
    c0^2 c2 (8 c2^3 e2 - 3 e3^2 - 8 e2 e4)) / (2 (c0 - c2) (c2^3 - e4))}
```

g3 is

In[101]:=
```
Simplify[Coefficient[Rem1, t, 3] + b11 ga31 + b12 ga32 + b13 ga33]
```

Out[101]=
$$\left\{\left(e1\,e3^2+e3^3-36\,c0\,c2^2\,e1\left(c2^3-e4\right)+12\,c0^2\,c2\,e3\left(c2^3-e4\right)-22\,e1\,e2\,e4-16\,e0\,e3\,e4+14\,e2\,e3\,e4+8\,e1\,e4^2+2\,c2^3\,(11\,e1\,e2+8\,e0\,e3-7\,e2\,e3-4\,e1\,e4)\right)\Big/\left(2\,(c0-c2)\left(c2^3-e4\right)\right)\right\}$$

In[102]:=
```
g3 := {(e1 e3^2 + e3^3 - 36 c0 c2^2 e1 (c2^3 - e4) +
    12 c0^2 c2 e3 (c2^3 - e4) - 22 e1 e2 e4 - 16 e0 e3 e4 + 14 e2 e3 e4 + 8 e1 e4^2 +
    2 c2^3 (11 e1 e2 + 8 e0 e3 - 7 e2 e3 - 4 e1 e4)) / (2 (c0 - c2) (c2^3 - e4))}
```

g4 is

In[103]:=
```
Simplify[Coefficient[Rem1, t, 4] + b11 ga41 + b12 ga42 + b13 ga43]
```

Out[103]=
$$\left\{\left(-8\,c2^6\,e0-4\,c0\,c2^5\,e2+16\,c0^2\,c2^4\,e4-4\,e2^2\,e4-16\,c0^2\,c2\,e4^2-2\,e4\left(8\,e1\,e3-3\,e3^2+4\,e0\,e4\right)+c2^3\left(4\,e2^2+16\,e1\,e3-5\,e3^2+16\,e0\,e4-12\,e2\,e4\right)+c0\,c2^2\left(-3\,e3^2+4\,e2\,e4\right)+e2\left(e3^2+12\,e4^2\right)\right)\Big/\left(2\,(c0-c2)\left(c2^3-e4\right)\right)\right\}$$

In[104]:=
```
g4 := {(-8 c2^6 e0 - 4 c0 c2^5 e2 + 16 c0^2 c2^4 e4 - 4 e2^2 e4 - 16 c0^2 c2 e4^2 -
    2 e4 (8 e1 e3 - 3 e3^2 + 4 e0 e4) + c2^3 (4 e2^2 + 16 e1 e3 - 5 e3^2 + 16 e0 e4 - 12 e2 e4) +
    c0 c2^2 (-3 e3^2 + 4 e2 e4) + e2 (e3^2 + 12 e4^2)) / (2 (c0 - c2) (c2^3 - e4))}
```

g5 is

In[105]:=
```
Simplify[Coefficient[Rem1, t, 5] + b11 ga51 + b12 ga52 + b13 ga53]
```

Out[105]=
$$\left\{\left(-10\,c2^6\,e1-2\,c0\,c2^5\,e3+e3^3+2\,c0\,c2^2\,e3\,e4-4\,e2\,e3\,e4-10\,e1\,e4^2+6\,e3\,e4^2+c2^3\,(4\,e2\,e3+20\,e1\,e4-6\,e3\,e4)\right)\Big/\left(2\,(c0-c2)\left(c2^3-e4\right)\right)\right\}$$

In[106]:=
```
g5 := {(-10 c2^6 e1 - 2 c0 c2^5 e3 + e3^3 + 2 c0 c2^2 e3 e4 - 4 e2 e3 e4 - 10 e1 e4^2 +
    6 e3 e4^2 + c2^3 (4 e2 e3 + 20 e1 e4 - 6 e3 e4)) / (2 (c0 - c2) (c2^3 - e4))}
```

In[107]:=
```
Cancel[Simplify[BT2]]
```

Out[107]=
$$\left\{\frac{1}{2\,(c0-c2)}\left(20\,c0^2\,c2\,e1-18\,c0\,c2^2\,e1+6\,c2^3\,e1-4\,e1\,e2+18\,c0^3\,e3-54\,c0^2\,c2\,e3+54\,c0\,c2^2\,e3-18\,c2^3\,e3-64\,c0^3\,c2^3\,t+96\,c0^2\,c2^4\,t-64\,c0\,c2^5\,t+16\,c2^6\,t-e1^2\,t+40\,c0^2\,c2\,e2\,t-36\,c0\,c2^2\,e2\,t+12\,c2^3\,e2\,t-4\,e2^2\,t-6\,e1\,e3\,t+24\,c0^3\,e4\,t-72\,c0^2\,c2\,e4\,t+72\,c0\,c2^2\,e4\,t-24\,c2^3\,e4\,t+22\,c0\,c2^2\,e1\,t^2-6\,c2^3\,e1\,t^2-4\,e1\,e2\,t^2+60\,c0^2\,c2\,e3\,t^2-54\,c0\,c2^2\,e3\,t^2+18\,c2^3\,e3\,t^2-12\,e2\,e3\,t^2-8\,e1\,e4\,t^2-96\,c0^2\,c2^4\,t^3+64\,c0\,c2^5\,t^3-16\,c2^6\,t^3+44\,c0\,c2^2\,e2\,t^3-12\,c2^3\,e2\,t^3-4\,e2^2\,t^3-6\,e1\,e3\,t^3-9\,e3^2\,t^3+80\,c0^2\,c2\,e4\,t^3-72\,c0\,c2^2\,e4\,t^3+24\,c2^3\,e4\,t^3-16\,e2\,e4\,t^3+8\,c2^3\,e1\,t^4+66\,c0\,c2^2\,e3\,t^4-18\,c2^3\,e3\,t^4-12\,e2\,e3\,t^4-8\,e1\,e4\,t^4-24\,e3\,e4\,t^4-64\,c0\,c2^5\,t^5+16\,c2^6\,t^5+16\,c2^3\,e2\,t^5-9\,e3^2\,t^5+88\,c0\,c2^2\,e4\,t^5-24\,c2^3\,e4\,t^5-16\,e2\,e4\,t^5-16\,e4^2\,t^5+24\,c2^3\,e3\,t^6-24\,e3\,e4\,t^6-16\,c2^6\,t^7+32\,c2^3\,e4\,t^7-16\,e4^2\,t^7\right)\right\}$$

In[108]:=
```
Rem2 := PolynomialRemainder[
  1/(2 (c0 - c2)) (20 c0^2 c2 e1 - 18 c0 c2^2 e1 + 6 c2^3 e1 - 4 e1 e2 + 18 c0^3 e3 - 54 c0^2 c2 e3 +
     54 c0 c2^2 e3 - 18 c2^3 e3 - 64 c0^3 c2^3 t + 96 c0^2 c2^4 t - 64 c0 c2^5 t + 16 c2^6 t - e1^2 t +
     40 c0^2 c2 e2 t - 36 c0 c2^2 e2 t + 12 c2^3 e2 t - 4 e2^2 t - 6 e1 e3 t + 24 c0^3 e4 t -
     72 c0^2 c2 e4 t + 72 c0 c2^2 e4 t - 24 c2^3 e4 t + 22 c0 c2^2 e1 t^2 - 6 c2^3 e1 t^2 -
     4 e1 e2 t^2 + 60 c0^2 c2 e3 t^2 - 54 c0 c2^2 e3 t^2 + 18 c2^3 e3 t^2 - 12 e2 e3 t^2 - 8 e1 e4 t^2 -
     96 c0^2 c2^4 t^3 + 64 c0 c2^5 t^3 - 16 c2^6 t^3 + 44 c0 c2^2 e2 t^3 - 12 c2^3 e2 t^3 - 4 e2^2 t^3 -
     6 e1 e3 t^3 - 9 e3^2 t^3 + 80 c0^2 c2 e4 t^3 - 72 c0 c2^2 e4 t^3 + 24 c2^3 e4 t^3 - 16 e2 e4 t^3 +
     8 c2^3 e1 t^4 + 66 c0 c2^2 e3 t^4 - 18 c2^3 e3 t^4 - 12 e2 e3 t^4 - 8 e1 e4 t^4 - 24 e3 e4 t^4 -
     64 c0 c2^5 t^5 + 16 c2^6 t^5 + 16 c2^3 e2 t^5 - 9 e3^2 t^5 + 88 c0 c2^2 e4 t^5 - 24 c2^3 e4 t^5 -
     16 e2 e4 t^5 - 16 e4^2 t^5 + 24 c2^3 e3 t^6 - 24 e3 e4 t^6 - 16 c2^6 t^7 + 32 c2^3 e4 t^7 - 16 e4^2 t^7),
  q0 + q1 t + q2 t^2 + q3 t^3 + q4 t^4 + q5 t^5 + q6 t^6, t]
```

Therefore h0 is

In[109]:=
```
Simplify[Coefficient[Rem2, t, 0] + b21 ga01 + b22 ga02 + b23 ga03]
```

Out[109]=
```
{(2 c0^5 c2 e1 - 2 c0^2 c2 e0 e1 - e1^3 + 4 e0 e1 e2 + 10 c0^6 e3 +
    e0^2 (-6 e1 + 10 e3) + c0^3 (6 e0 e1 - 4 e1 e2 - 20 e0 e3)) / (2 (c0 - c2) (c0^3 - e0))}
```

In[110]:=
```
h0 := {(2 c0^5 c2 e1 - 2 c0^2 c2 e0 e1 - e1^3 + 4 e0 e1 e2 + 10 c0^6 e3 +
    e0^2 (-6 e1 + 10 e3) + c0^3 (6 e0 e1 - 4 e1 e2 - 20 e0 e3)) / (2 (c0 - c2) (c0^3 - e0))}
```

h1 is

In[111]:=
```
Simplify[Coefficient[Rem2, t, 1] + b21 ga11 + b22 ga12 + b23 ga13]
```

Out[111]=
```
{(-16 c0^4 c2^2 e0 + 16 c0 c2^2 e0^2 + 4 c0^5 c2 e2 - e1^2 e2 + c0^2 c2 (3 e1^2 - 4 e0 e2) +
    e0 (-6 e1^2 + 4 e2^2 + 16 e1 e3) + 8 c0^6 e4 + e0^2 (-12 e2 + 8 e4) +
    c0^3 (5 e1^2 + 12 e0 e2 - 4 e2^2 - 16 e1 e3 - 16 e0 e4)) / (2 (c0 - c2) (c0^3 - e0))}
```

In[112]:=
```
h1 := {(-16 c0^4 c2^2 e0 + 16 c0 c2^2 e0^2 + 4 c0^5 c2 e2 - e1^2 e2 + c0^2 c2 (3 e1^2 - 4 e0 e2) +
    e0 (-6 e1^2 + 4 e2^2 + 16 e1 e3) + 8 c0^6 e4 + e0^2 (-12 e2 + 8 e4) +
    c0^3 (5 e1^2 + 12 e0 e2 - 4 e2^2 - 16 e1 e3 - 16 e0 e4)) / (2 (c0 - c2) (c0^3 - e0))}
```

h2 is

In[113]:=
```
Simplify[Coefficient[Rem2, t, 2] + b21 ga21 + b22 ga22 + b23 ga23]
```

Out[113]=
```
{-(12 c0^4 c2^2 e1 - 12 c0 c2^2 e0 e1 + e1^3 - 36 c0^5 c2 e3 +
    36 c0^2 c2 e0 e3 + e1^2 e3 + 2 e0 (4 e0 - 11 e2) e3 + 2 e0 e1 (7 e2 - 8 e4) -
    2 c0^3 (7 e1 e2 + 4 e0 e3 - 11 e2 e3 - 8 e1 e4)) / (2 (c0 - c2) (c0^3 - e0))}
```

In[114]:=
```
h2 := {-(12 c0^4 c2^2 e1 - 12 c0 c2^2 e0 e1 + e1^3 - 36 c0^5 c2 e3 +
    36 c0^2 c2 e0 e3 + e1^2 e3 + 2 e0 (4 e0 - 11 e2) e3 + 2 e0 e1 (7 e2 - 8 e4) -
    2 c0^3 (7 e1 e2 + 4 e0 e3 - 11 e2 e3 - 8 e1 e4)) / (2 (c0 - c2) (c0^3 - e0))}
```

h3 is

In[115]:=
```
Simplify[Coefficient[Rem2, t, 3] + b21 ga31 + b22 ga32 + b23 ga33]
```

Out[115]=
$$\{(16\,c2^3\,e0^2 - 8\,c0^4\,c2^2\,e2 - e1^2\,e2 - 8\,e0\,e2^2 + c0\,c2^2\,(3\,e1^2 + 8\,e0\,e2) - 8\,e0\,e1\,e3 + 19\,e0\,e3^2 + 32\,c0^5\,c2\,e4 - 32\,c0^2\,c2\,e0\,e4 - 16\,e0^2\,e4 - e1^2\,e4 + 24\,e0\,e2\,e4 + c0^3\,(-16\,c2^3\,e0 + 8\,e2^2 + 8\,e1\,e3 - 19\,e3^2 + 16\,e0\,e4 - 24\,e2\,e4))) / (2\,(c0 - c2)\,(c0^3 - e0))\}$$

In[116]:=
```
h3 := {(16 c2^3 e0^2 - 8 c0^4 c2^2 e2 - e1^2 e2 - 8 e0 e2^2 + c0 c2^2 (3 e1^2 + 8 e0 e2) - 8 e0 e1 e3 +
      19 e0 e3^2 + 32 c0^5 c2 e4 - 32 c0^2 c2 e0 e4 - 16 e0^2 e4 - e1^2 e4 + 24 e0 e2 e4 +
      c0^3 (-16 c2^3 e0 + 8 e2^2 + 8 e1 e3 - 19 e3^2 + 16 e0 e4 - 24 e2 e4))) / (2
      (c0 - c2) (c0^3 - e0))}
```

h4 is

In[117]:=
```
Simplify[Coefficient[Rem2, t, 4] + b21 ga41 + b22 ga42 + b23 ga43]
```

Out[117]=
$$\{-(-14\,c2^3\,e0\,e1 - 26\,c0^4\,c2^2\,e3 + 26\,c0\,c2^2\,e0\,e3 + e1^2\,e3 + 8\,e0\,e2\,e3 + 14\,e0\,e1\,e4 - 42\,e0\,e3\,e4 + 2\,c0^3\,(7\,c2^3\,e1 - 4\,e2\,e3 - 7\,e1\,e4 + 21\,e3\,e4))) / (2\,(c0 - c2)\,(c0^3 - e0))\}$$

In[118]:=
```
h4 :=
 {-(-14 c2^3 e0 e1 - 26 c0^4 c2^2 e3 + 26 c0 c2^2 e0 e3 + e1^2 e3 + 8 e0 e2 e3 + 14 e0 e1 e4 - 42 e0
      e3 e4 + 2 c0^3 (7 c2^3 e1 - 4 e2 e3 - 7 e1 e4 + 21 e3 e4)) / (2 (c0 - c2) (c0^3 - e0))}
```

h5 is

In[119]:=
```
Simplify[Coefficient[Rem2, t, 5] + b21 ga51 + b22 ga52 + b23 ga53]
```

Out[119]=
$$\{(c2^3\,(e1^2 + 12\,(-c0^3 + e0)\,e2) + e0\,e3^2 + 24\,c0\,c2^2\,(c0^3 - e0)\,e4 - e1^2\,e4 - 12\,e0\,e2\,e4 + 24\,e0\,e4^2 - c0^3\,(e3^2 - 12\,e2\,e4 + 24\,e4^2))) / (2\,(c0 - c2)\,(c0^3 - e0))\}$$

In[120]:=
```
h5 := {(c2^3 (e1^2 + 12 (-c0^3 + e0) e2) + e0 e3^2 + 24 c0 c2^2 (c0^3 - e0) e4 - e1^2 e4 -
      12 e0 e2 e4 + 24 e0 e4^2 - c0^3 (e3^2 - 12 e2 e4 + 24 e4^2)) / (2 (c0 - c2) (c0^3 - e0))}
```

In[121]:=
```
f1 := 2 (c0 - c2) (c0^3 - e0) (c2^3 - e4) (5 h0 - g1)
```

In[122]:=
```
f2 := 2 (c0 - c2) (c0^3 - e0) (c2^3 - e4) (2 h1 - g2)
```

In[123]:=
```
f3 := 2 (c0 - c2) (c0^3 - e0) (c2^3 - e4) (h2 - g3)
```

In[124]:=
```
f4 := 2 (c0 - c2) (c0^3 - e0) (c2^3 - e4) (h3 - 2 g4)
```

In[125]:=
```
f5 := 2 (c0 - c2) (c0^3 - e0) (c2^3 - e4) (h4 - 5 g5)
```

In[126]:=
```
Collect[f1, {e1, e3}, Simplify]
```

Out[126]=
$$\{e1\,((-c0^3 + e0)\,e3^2 + 12\,(c0^3 - e0)\,(3\,c0^2\,c2 - e0 - e2)\,(c2^3 - e4)) + 36\,(c0^3 - e0)^2\,e3\,(c2^3 - e4) + e1^3\,(-5\,c2^3 + 5\,e4)\}$$

In[127]:=
```
Collect[f2, {e1, e3}, Simplify]
```

Out[127]=
$$\{(c0^3 - e0)\,(3\,c0^2\,c2 - e0 - e2)\,e3^2 - e1^2\,(9\,c0^3 - 6\,c0^2\,c2 - 7\,e0 + 2\,e2)\,(c2^3 - e4) - 24\,(c0^3 - e0)\,e1\,e3\,(c2^3 - e4)\}$$

In[128]:=
```
Collect[f3, {e1, e3}, Simplify]
```

Out[128]=
$$\{(-c0^3 + e0)\,e3^3 + e1\,((-c0^3 + e0)\,e3^2 + 8\,(c0^3 - e0)\,(c2^3 - e4)\,(3\,c0\,c2^2 - e2 - e4)) + 8\,(c0^3 - e0)\,(3\,c0^2\,c2 - e0 - e2)\,e3\,(c2^3 - e4) + e1^3\,(-c2^3 + e4) + e1^2\,e3\,(-c2^3 + e4)\}$$

In[129]:=
```
Collect[f4, {e1, e3}, Simplify]
```

Out[129]=
$$\{-24\,(c0^3-e0)\,e1\,e3\,(c2^3-e4)+e1^2\,(c2^3-e4)\,(3\,c0\,c2^2-e2-e4)+(c0^3-e0)\,e3^2\,(6\,c0\,c2^2-9\,c2^3-2\,e2+7\,e4)\}$$

In[130]:=
```
Collect[f5, {e1, e3}, Simplify]
```

Out[130]=
$$\{(-5\,c0^3+5\,e0)\,e3^3+36\,(c0^3-e0)\,e1\,(c2^3-e4)^2+12\,(c0^3-e0)\,e3\,(c2^3-e4)\,(3\,c0\,c2^2-e2-e4)+e1^2\,e3\,(-c2^3+e4)\}$$

In[131]:=
```
Collect[f2 - f4, {e1, e3}, Simplify]
```

Out[131]=
$$\{(c0^3-e0)\,e3^2\,(3\,c0^2\,c2-6\,c0\,c2^2+9\,c2^3-e0+e2-7\,e4)-e1^2\,(c2^3-e4)\,(9\,c0^3-6\,c0^2\,c2+3\,c0\,c2^2-7\,e0+e2-e4)\}$$

In[132]:=
```
Simplify[Cancel[((c0^3 - e0) (3 c0^2 c2 - 6 c0 c2^2 + 9 c2^3 - e0 + e2 - 7 e4))
    ((c0^3 - e0) (3 c0^2 c2 - e0 - e2) e3^2 - e1^2 (9 c0^3 - 6 c0^2 c2 - 7 e0 + 2 e2) (c2^3 - e4))^2 -
   (24 (c0^3 - e0) e1 e3 (c2^3 - e4))^2 /.
  {e1 -> 1, e3 -> Sqrt[((c2^3 - e4) (9 c0^3 - 6 c0^2 c2 + 3 c0 c2^2 - 7 e0 + e2 - e4)) /
      ((c0^3 - e0) (3 c0^2 c2 - 6 c0 c2^2 + 9 c2^3 - e0 + e2 - 7 e4))]}]]
```

Out[132]=
$$\left(9\,(c0^3-e0)\,(c2^3-e4)^2\left(-64\,(c2^3-e4)\,(9\,c0^3-6\,c0^2\,c2+3\,c0\,c2^2-7\,e0+e2-e4)+(18\,c0^4\,c2^2+3\,c2^3\,(7\,e0-2\,e2)+5\,e0\,e2-e2^2+3\,c0\,c2^2\,(-5\,e0+e2)-3\,c0^3\,(12\,c2^3+2\,e2-7\,e4)+3\,c0^2\,c2\,(6\,c2^3+e2-5\,e4)-16\,e0\,e4+5\,e2\,e4)^2\right)\right)/(3\,c0^2\,c2-6\,c0\,c2^2+9\,c2^3-e0+e2-7\,e4)$$

In[133]:=
```
Collect[
  Expand[(-64 (c2^3 - e4) (9 c0^3 - 6 c0^2 c2 + 3 c0 c2^2 - 7 e0 + e2 - e4) + (18 c0^4 c2^2 + 3 c2^3
        (7 e0 - 2 e2) + 5 e0 e2 - e2^2 + 3 c0 c2^2 (-5 e0 + e2) - 3 c0^3 (12 c2^3 + 2 e2 - 7 e4) +
      3 c0^2 c2 (6 c2^3 + e2 - 5 e4) - 16 e0 e4 + 5 e2 e4)^2)], e0, Simplify]
```

Out[133]=
$$\begin{aligned}&324\,c0^8\,c2^4+36\,c2^6\,e2^2+e2^4+9\,c0^6\,(216\,c2^6+12\,c2^3\,(5\,e2-19\,e4)+(2\,e2-7\,e4)^2)-\\&108\,c0^7\,c2^2\,(12\,c2^3+2\,e2-7\,e4)+64\,e2\,e4-10\,e2^3\,e4-64\,e4^2+25\,e2^2\,e4^2+\\&e0^2\,(15\,c0\,c2^2-21\,c2^3-5\,e2+16\,e4)^2+4\,c2^3\,(-16\,e2+3\,e2^3+16\,e4-15\,e2^2\,e4)-\\&6\,c0\,c2^2\,(e2^3+c2^3\,(32+6\,e2^2)-32\,e4-5\,e2^2\,e4)-\\&18\,c0^5\,c2\,(72\,c2^6+2\,e2^2+6\,c2^3\,(3\,e2-17\,e4)-17\,e2\,e4+35\,e4^2)+\\&9\,c0^4\,c2^2\,(36\,c2^6-7\,e2^2+24\,e2\,e4+25\,e4^2-12\,c2^3\,(3\,e2+5\,e4))+\\&6\,c0^3\,(90\,c2^6\,e2+2\,e2^3+96\,e4-17\,e2^2\,e4+35\,e2\,e4^2+3\,c2^3\,(-32+9\,e2^2-39\,e2\,e4))-\\&2\,e0\,(-378\,c0^2\,c2^7+5\,e2^3-153\,c0\,c2^5\,(6\,c0^3+e2)+18\,c2^6\,(57\,c0^3+7\,e2)-3\,c0\,c2^2\\&(3\,c0^3\,(20\,e2-67\,e4)+e2\,(10\,e2-41\,e4))+224\,e4-41\,e2^2\,e4+80\,e2\,e4^2+9\,c0^2\,c2^4\\&(30\,c0^3-12\,e2+67\,e4)+c2^3\,(-224+51\,e2^2+27\,c0^3\,(13\,e2-46\,e4)-201\,e2\,e4)-\\&3\,c0^2\,c2\,(5\,e2^2-41\,e2\,e4+80\,e4^2)+3\,c0^3\,(10\,e2^2-67\,e2\,e4+112\,e4^2))-3\,c0^2\,c2\\&(72\,c2^6\,e2+c2^3\,(-128+21\,e2^2-120\,e2\,e4)+2\,(e2^3+64\,e4-10\,e2^2\,e4+25\,e2\,e4^2))\end{aligned}$$

In[134]:=
```
Simplify[Coefficient[(-64 (c2^3 - e4) (9 c0^3 - 6 c0^2 c2 + 3 c0 c2^2 - 7 e0 + e2 - e4) +
    (18 c0^4 c2^2 + 3 c2^3 (7 e0 - 2 e2) + 5 e0 e2 - e2^2 + 3 c0 c2^2 (-5 e0 + e2) - 3 c0^3
      (12 c2^3 + 2 e2 - 7 e4) + 3 c0^2 c2 (6 c2^3 + e2 - 5 e4) - 16 e0 e4 + 5 e2 e4)^2), e0, 2]]
```

Out[134]=
```
(15 c0 c2^2 - 21 c2^3 - 5 e2 + 16 e4)^2
```

In[135]:=
```
Simplify[Coefficient[(-64 (c2^3 - e4) (9 c0^3 - 6 c0^2 c2 + 3 c0 c2^2 - 7 e0 + e2 - e4) +
    (18 c0^4 c2^2 + 3 c2^3 (7 e0 - 2 e2) + 5 e0 e2 - e2^2 + 3 c0 c2^2 (-5 e0 + e2) - 3 c0^3
      (12 c2^3 + 2 e2 - 7 e4) + 3 c0^2 c2 (6 c2^3 + e2 - 5 e4) - 16 e0 e4 + 5 e2 e4)^2), e0, 1]]
```

Out[135]=
```
-2 (-378 c0^2 c2^7 + 5 e2^3 - 153 c0 c2^5 (6 c0^3 + e2) +
   18 c2^6 (57 c0^3 + 7 e2) - 3 c0 c2^2 (3 c0^3 (20 e2 - 67 e4) + e2 (10 e2 - 41 e4)) +
   224 e4 - 41 e2^2 e4 + 80 e2 e4^2 + 9 c0^2 c2^4 (30 c0^3 - 12 e2 + 67 e4) +
   c2^3 (-224 + 51 e2^2 + 27 c0^3 (13 e2 - 46 e4) - 201 e2 e4) -
   3 c0^2 c2 (5 e2^2 - 41 e2 e4 + 80 e4^2) + 3 c0^3 (10 e2^2 - 67 e2 e4 + 112 e4^2))
```

In[136]:=
```
Simplify[Coefficient[(-64 (c2^3 - e4) (9 c0^3 - 6 c0^2 c2 + 3 c0 c2^2 - 7 e0 + e2 - e4) +
    (18 c0^4 c2^2 + 3 c2^3 (7 e0 - 2 e2) + 5 e0 e2 - e2^2 + 3 c0 c2^2 (-5 e0 + e2) - 3 c0^3
      (12 c2^3 + 2 e2 - 7 e4) + 3 c0^2 c2 (6 c2^3 + e2 - 5 e4) - 16 e0 e4 + 5 e2 e4)^2), e0, 0]]
```

Out[136]=
```
324 c0^8 c2^4 + 36 c2^6 e2^2 + e2^4 + 9 c0^6 (216 c2^6 + 12 c2^3 (5 e2 - 19 e4) + (2 e2 - 7 e4)^2) -
 108 c0^7 c2^2 (12 c2^3 + 2 e2 - 7 e4) + 64 e2 e4 - 10 e2^3 e4 -
 64 e4^2 + 25 e2^2 e4^2 + 4 c2^3 (-16 e2 + 3 e2^3 + 16 e4 - 15 e2^2 e4) -
 6 c0 c2^2 (e2^3 + c2^3 (32 + 6 e2^2) - 32 e4 - 5 e2^2 e4) -
 18 c0^5 c2 (72 c2^6 + 2 e2^2 + 6 c2^3 (3 e2 - 17 e4) - 17 e2 e4 + 35 e4^2) +
 9 c0^4 c2^2 (36 c2^6 - 7 e2^2 + 24 e2 e4 + 25 e4^2 - 12 c2^3 (3 e2 + 5 e4)) + 6 c0^3
  (90 c2^6 e2 + 2 e2^3 + 96 e4 - 17 e2^2 e4 + 35 e2 e4^2 + 3 c2^3 (-32 + 9 e2^2 - 39 e2 e4)) - 3 c0^2
  c2 (72 c2^6 e2 + c2^3 (-128 + 21 e2^2 - 120 e2 e4) + 2 (e2^3 + 64 e4 - 10 e2^2 e4 + 25 e2 e4^2))
```